## SOLUTION OF EXERCISE # 6.1

# Exercise # 6.1

**Q.1:** If $\vec{a} = 3i - j - 4k$, $\vec{b} = -2i + 4j - 3k$ and $\vec{c} = i + 2j - k$. Find unit vector parallel to $3\vec{a} - 2\vec{b} + 4\vec{c}$.
(IA-2019), (IIA-2019)

**Sol.** Let $\vec{v} = 3\vec{a} - 2\vec{b} + 4\vec{c}$

$= 3(3i - j - 4k) - 2(-2i + 4j - 3k) + 4(i + 2j - k)$

$\vec{v} = 9i - 3j - 12k + 4i - \cancel{8j} + 6k + 4i + \cancel{8j} - 4k$

$\vec{v} = 17i - 3j - 10k$

$|\vec{v}| = \sqrt{(17)^2 + (-3)^2 + (-10)^2} = \sqrt{289 + 9 + 100} = \sqrt{398}$

Unit Vector $= \hat{v} = \dfrac{\vec{v}}{|\vec{v}|} = \boxed{\dfrac{17i - 3j - 10k}{\sqrt{398}}}$

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**Q.2:** Find the vector whose magnitude is 5 and which is in the direction of the vector 4i – 3j + k? (IIA-2017)

**Sol.** Let $\vec{a}$ be a require vector.

So $|\vec{a}| = 5$

& Let $\vec{b} = 4i - 3j + k$

$|\vec{b}| = \sqrt{(4)^2 + (-3)^2 + (1)^2} = \sqrt{16 + 9 + 1} = \sqrt{26}$

As $\vec{a}$ & $\vec{b}$ have same direction

So $\hat{a} = \hat{b}$

$$\frac{\vec{a}}{|\vec{a}|} = \frac{\vec{b}}{|\vec{b}|}$$

$$\frac{\vec{a}}{5} = \frac{4i - 3j + k}{\sqrt{26}} \Rightarrow \vec{a} = \boxed{\frac{5(4i - 3j + k)}{\sqrt{26}}}$$

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**Q.3:** For what value of 'm', the vectors 4i + 2j – 3k and mi – j + $\sqrt{3}$k have same magnitude? (IIA-2021)

**Sol.** Here m = ?

Let $\vec{a} = 4i + 2j - 3k$ & $\vec{b} = mi - j + \sqrt{3}k$

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As, $\vec{a}$ & $\vec{b}$ have same magnitude

so. $|\vec{a}| = |\vec{b}|$

$$\sqrt{(4)^2+(2)^2+(-3)^2} = \sqrt{(m)^2+(-1)^2+(\sqrt{3})^2}$$

$$\sqrt{16+4+9} = \sqrt{m^2+1+3}$$

$$\sqrt{29} = \sqrt{m^2+4}$$

Squaring both sides

$$(\sqrt{29})^2 = (\sqrt{m^2+4})^2$$

$$29 = m^2 + 4$$

$$29 - 4 = m^2$$

$$25 = m^2$$

$$\pm\sqrt{25} = \sqrt{m^2} \Rightarrow \boxed{m = \pm 5}$$

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**Q.4:** Given the points A = (1, 2, −1), B = (−3, 1, 2) and C = (0, −4, 3)

**(i)** Find, $\overline{AB}$, $\overline{BC}$, $\overline{AC}$? (IIA-2021)

**Sol.** $\overline{AB} = [-3,1,2] - [1,2,-1] = [-4, -1, 3]$

$\overline{BC} = [0, -4, 3] - [-3,1,2] = [3, -5, 1]$

$\overline{AC} = [0, -4,3] - [1,2,-1] = [-1, -6, 4]$

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**(ii)** Show that: $\overline{AB} + \overline{BC} = \overline{AC}$?

**Sol.** L.H.S. $= \overline{AB} + \overline{BC}$

$= [-4, -1, 3] + [3, -5, 1]$

$= [-1, -6, 4] = \overline{AC}$ = R.H.S. **Proved**

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**Q.5:** Find the length of the sides of a triangle, whose vertices are A = (2, 4, −1), B = (4, 5, 1) and C = (3, 6, −3) and show that the triangle is right angled.

**Sol.** $\overline{AB} = [4, 5, 1] - [2, 4, -1] = [2, 1, 2]$

$\overline{BC} = [3, 6, -3] - [4, 5, 1] = [-1, 1, -4]$

$\overline{AC} = [3, 6, -3] - [2, 4, -1] = [1, 2, -2]$

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$$|\overrightarrow{AB}| = \sqrt{(2)^2+(1)^2+(2)^2} = \sqrt{4+1+4} = \sqrt{9} = 3$$

$$|\overrightarrow{BC}| = \sqrt{(-1)^2+(1)^2+(-4)^2} = \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}$$

$$|\overrightarrow{AC}| = \sqrt{(1)^2+(2)^2+(-2)^2} = \sqrt{1+4+4} = \sqrt{9} = 3$$

Now by Pythagoras theorem

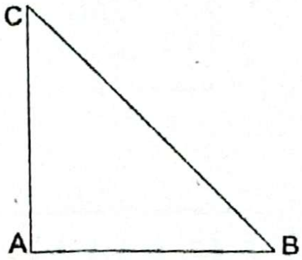
$$|\overrightarrow{AB}|^2 + |\overrightarrow{AC}|^2 = |\overrightarrow{BC}|^2$$

$$(3)^2+(3)^2 = (3\sqrt{2})^2$$

$$9+9 = 9(2)$$

$$18 = 18$$

L.H.S. = R.H.S.




Hence ABC is a right angle triangle.

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**Q.6: If vectors 3i + j − k and λi − 4j + 4k are parallel, find the value of λ?**

**Sol.** Same as Q.12 of Ex# 8.2 (see page # 261)

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**Q7: Show that the vectors $4i-6j+9k$ and $-6i+9j-\frac{27}{2}k$ are parallel.** (IIA-2017), (IA-2018)

**Sol.** Let $\vec{a} = 4i-6j+9k,\ \vec{b} = -6i+9j-\frac{27}{2}k$

$$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 4 & -6 & 9 \\ -6 & 9 & -27/2 \end{vmatrix}$$

$$= i\begin{vmatrix} -6 & 9 \\ 9 & -27/2 \end{vmatrix} - j\begin{vmatrix} 4 & 9 \\ -6 & -27/2 \end{vmatrix} + k\begin{vmatrix} 4 & -6 \\ -6 & 9 \end{vmatrix}$$

$$= i\left(-6\left(-27/2\right)-81\right) - j\left(4\left(-27/2\right)-(-54)\right) + k(36-36)$$

$$= i(81-81) - j(-54+54) + k(36-36)$$

$$= i(0)-j(0)+k(0) = \boxed{0}$$ Hence $\vec{a}$ & $\vec{b}$ are parallel.

## SOLUTION OF EXERCISE # 6.1

**Q.8: Find real numbers x,y and z such that**

**[a]** $7xi + (y-3)j + 6k = 10i + 8j - 3zk$ **(IA-2021)**

**Sol.** Comparing coefficient of i, j, & k, we have:

| | | |
|---|---|---|
| $7x = 10$ | $y - 3 = 8$ | $6 = -3z$ |
| $\boxed{x = \frac{10}{7}}$ | $y = 8 + 3$ | $3z = -6$ |
| | $\boxed{y = 11}$ | $z = -\frac{6}{3}$ |
| | | $\boxed{z = -2}$ |

**[b]** $(x+4)i + (y-5)j + (z-1)k = 0.$

**Sol.** $(x+4)i + (y-5)j + (z-1)k = 0i+0j+0k$

Comparing coefficients of i, j, & k, we have:

| | | |
|---|---|---|
| $x + 4 = 0$ | $y - 5 = 0$ | $z - 1 = 0$ |
| $\boxed{x = -4}$ | $\boxed{y = 5}$ | $\boxed{z = 1}$ |

**Q.9: Given the vectors $\vec{a} = 3i - 2j + 4k$ and $\vec{b} = 2i + j + 3k$. Find the magnitude and direction cosines of:**

**(i)** $\vec{a} - \vec{b}$ **(IIA-2017)**

**Sol.** Let $\vec{v} = \vec{a} - \vec{b} = (3i - 2j + 4k) - (2i + j + 3k)$

$\vec{v} = 3i - 2j + 4k - 2i - j - 3k$

$\vec{v} = i - 3j + k$

| | |
|---|---|
| Magnitude $= \lvert\vec{v}\rvert$ | Direction cosines: |
| $= \sqrt{(1)^2 + (-3)^2 + (1)^2}$ | $\boxed{\frac{1}{\sqrt{11}}, -\frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}}}$ |
| $= \sqrt{1+9+1} = \boxed{\sqrt{11}}$ | |

**(ii)** $3\vec{a} - 2\vec{b}$ **(IA-2016), (IIA-2016)**

**Sol.** Let $\vec{v} = 3\vec{a} - 2\vec{b}$

$\vec{v} = 3(3i - 2j + 4k) - 2(2i + j + 3k)$

$\vec{v} = 9i - 6j + 12k - 4i - 2j - 6k$

$\vec{v} = 5i - 8j + 6k$

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Magnitude $= |\vec{v}|$

$= \sqrt{(5)^2 + (-8)^2 + (6)^2}$

$= \sqrt{25 + 64 + 36}$

$= \sqrt{125} = \sqrt{25 \times 5} = \boxed{5\sqrt{5}}$

Direction cosines:

$\boxed{\frac{5}{5\sqrt{5}}, \frac{-8}{5\sqrt{5}}, \frac{6}{5\sqrt{5}}}$

**Q.10: If the position vectors of $\vec{A}$ and $\vec{B}$ are $5i - 2j + 4k$ and $i + 3j + 7k$ respectively, find the magnitude and direction cosines of $\overrightarrow{AB}$.**

**Sol.** Here P.V. of $\vec{A} = 5i - 2j + 4k$

And P.V. of $\vec{B} = i + 3j + 7k$

$\overrightarrow{AB}$ = P.V. of $\vec{B}$ – P.V of $\vec{A}$

$\overrightarrow{AB} = (i + 3j + 7k) - (5i - 2j + 4k)$

$\overrightarrow{AB} = i + 3j + 7k - 5i + 2j - 4k$

$\overrightarrow{AB} = -4i + 5j + 3k$

Magnitude $= |\overrightarrow{AB}|$

$= \sqrt{(-4)^2 + (5)^2 + (3)^2}$

$= \sqrt{16 + 25 + 9} = \boxed{\sqrt{50}}$

Direction cosines :

$\boxed{\frac{-4}{\sqrt{50}}, \frac{5}{\sqrt{50}}, \frac{3}{\sqrt{50}}}$